Written Exam for the B.Sc. or M.Sc. in Economics summer 2013

Pricing Financial Assets

Final Exam

June 19, 2013

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 2 pages in total including this page.

The Exam consists of 3 problems that will enter the evaluation with equal weights.

1. Let the price of a traded stock, S, paying no dividends be modelled by the geometric Brownian motion

$$dS = \mu S dt + \sigma S dz$$

where μ and σ are constants, and where dt and dz are the standard short hand notations for a small time-step and a Brownian increment.

Consider a forward contract on the stock with a time to maturity of T. Let the current ($t = 0$) price of the stock be S_0, and assume that the risk-free rate of interest is a constant r.

(a) What is relationship between the forward price F_0 at $t = 0$ and the stock price? Comment on the result.

(b) Use Ito's lemma derive the process followed by the forward price F.

(c) Compare the price of a European call option on the stock with the price of a European call option on the forward when the two options have the same strikes and maturities.

2. (a) What is a ratings transitions matrix? What is the typical structure af such a matrix?

(b) What assumptions are needed to use the matrix to derive an estimate of the likelihood of a default of a rated issuer within a certain time horizon? And will this estimate be a real world or a risk neutral probability?

(c) Suppose the ratings transitions matrix is used to determine the default probability of two rated issuers. Describe in general terms how a Copula function can be used to assess the probability of a default of both issuers within a certain time horizon.

3. Consider an interest rate cap with a life of T, a principal of L, and a cap rate of R_K. Consider reset dates $0 = t_0 < t_1 < t_2, \ldots < t_n$, and let R_k be the Libor rate for the period from t_k to t_{k+1} known at t_k.

(a) Describe the payments of the cap, and define a caplet.

(b) Show that the cap can be considered as a portfolio of European put options on zero-coupon bonds.

(c) A standard market practice is to price a caplet (at $t = 0$) with the Black formula:

$$Caplet^{Black}(0, k, L, R_K, \sigma_k) = LP(0, t_{k+1})\tau_k(F_k N(d_1) - R_K N(d_2))$$

with

$$d_1 = \frac{ln(F_k/R_K) + 0.5\sigma_k^2 t_k}{\sigma_k \sqrt{t_k}}$$
$$d_2 = d_1 - \sigma_k \sqrt{t_k}$$

where $P(t, T)$ is the price of a zero coupon bond at t maturing at T, F_k is the forward rate at 0 for the time interval (t_k, t_{k+1}) with length τ_k, and σ_k the volatility of this rate. What assumptions can justify this formula?